UNIT-I

Numerical Methods

Maharaja Agrasen University, Baddi (HP)

Subject: Numerical Methods

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❖ Solutions of Algebraic and transcendental equations

For these problems numerical methods are available. Numerical methods have always been useful and their role in engineering design and scientific research is of fundamental importance, because they can give a solution for a problem when ordinary analytical methods fail.

Definition: An equation $f(x) = 0$ where

$f(x) = a_o x^n + a_1 x^{n-1} \ldots\ldots a_n$ where $a_o, a_1 \ldots\ldots a_n$

are real numbers $a_o \neq 0$, is called an n^{th} degree algebraic equation.

If $f(x)$ contains some other functions namely trigonometric logarithmic exponential etc, then the equation $f(x) = 0$ is called transcendental equation.

Root : A real number α is called the real root of the equation $f(x)=0$ if and only if $f(\alpha)=0$.

Geometrically the real root of $f(x)=0$ is the value of x at which the graph of $f(x)$ meets the x- axis. We can find the roots of an algebraic or transcendental equation by using numerical methods.

❖ Method 1: Bisection method

If a function f(x) is continuous b/w x_0 and x_1 and $f(x_0)$ & $f(x_1)$ are of opposite signs, then there exsist at least one root b/w x_0 and x_1

- Let $f(x_0)$ be –ve and $f(x_1)$ be +ve ,then the root lies b/w xo and x_1 and its approximate value is given by $x_2=(x_0+x_1)/2$
- If $f(x_2)=0$,we conclude that x_2 is a root of the equ $f(x)=0$
- Otherwise the root lies either b/w x_2 and x_1 (or) b/w x_2 and x_0 depending on wheather $f(x_2)$ is +ve or –ve.

- Find the real root of $x^3 - x - 1 = 0$ using bisection method.

Solution: $f(x) = x^3 - x - 1$

$f(1) = -1 < 0$, $f(2) = 5 > 0$

root lies between 1 and 1.5

$f(1.5) = 0.875 > 0$

Root lies between 1.5 and 1.25.

X_3 = 1+1.5/2 = 1.375

f(1.375) = 0.2246 > 0

root lies between 1.25 and 1.375

X_4 = 1.25 + 1.375 /2 =1.3125

F (1.3125) = -0.015 < 0

Root lies between 1.3125 and 1.3175

X_5 = 1.3125 + 1.375/2 = 1.3437

f(1.3437) = 0.0823 > 0

Root lies between 1.3437 and 1.3125

X_6 = 1.3437 + 1.3125/2 1.3281

f(1.3281) = 0.0144 > 0

Root lies between 1.3125 and 1.3281.

X_7 = 1.3125 + 1.3281 /2 = 1.3203

f(1.3203 = -0.018 < 0

Root lies between 1.3203 and 1.3281

X_8= 1.3203 + 1.3281/2 = 1.3242

Therefore root = 1.3242

❖ Method 2: Iteration method or successive approximation

Consider the equation $f(x)=0$ which can take in the form

$x = ø(x)$ -------------(1) where $|ø^1(x)|<1$ for all values of x.

Taking initial approximation is x_0

we put $x_1=ø(x_0)$ and take x_1 is the first approximation

$x_2=ø(x_1)$, x_2 is the second approximation

$x_3=ø(x_2)$,x_3 is the third approximation

.

$x_n=ø(x_{n-1})$,x_n is the n^{th} approximation

Such a process is called an iteration process.

- Find the root of the equation cosx = 3x-1 correct to 4 decimals using iterative method.

Solution : cosx = 3x-1

$f(x) = \cos x - 3x + 1$

$f(0) = 2$

$f(0.5) = 0.3375$

$f(0.6) = 0.025 > 0$

$f(0.7) = -0.335 < 0$

Root lies between 0.6 and 0.7

$\cos x = 3x-1$

$3x=1 + \cos x$

$x = 1 + \cos x /3$

$\Phi (x) = -\sin x /3$

$|\Phi (x)| = | -\sin x /3 | < 1.$

Iterative method is applicable and the iterative formula is x_{n+1}

$= \Phi (x_n)$

$x_{n+1} = 1 + \cos x_n /3$

put n= 0, $x_o = 0.6$

$x_1 = 1 + \cos x_0 /3 = 0.6084$

$x_2 = 1 + \cos x_1/3 = 0.6068$

$x_3 = 1 + \cos x_2/3 = 0.6071$

$x_4 = 1 + \cos x_3/3 = o.6071$

Therefore in two successive iterations we get the value of x is 0.6071 i.e x_3 and x_4 have equal values.

Therefore root = 0.6071 .